The I Tjing of music

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Abstract

This treatise on the phenomenology of music is based on epimore interval relationships.
The impetus to describe a phenomenology of music arises from my previous research into the structure of keyboard planimetries.

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EPIMORIC RATIOS

Overtones and undertones

With reference to Helmholtz's book "Sensations of Tone" we can assume that every frequency has both an overtone series and an undertone series. If we set the value of a randomly selected fundamental frequency to 1, then each overtone satisfies the formula (1+x) and each undertone satisfies the formula 1/(1+x), where x always represents a positive integer natural number.

Epimoric ratios as the primordial source of the phenomenon of music

The starting point of this review is that as soon as we expand our consideration from one to several tones, it appears that there are interchangeable concepts, a relativism that is inherent to the mobility of music. A fundamental tone together with any overtone forms an interval: fundamental and overtone. The fundamental tone is an undertone of the overtone. The concept of the fundamental tone in music is at most an initial situation, since in the course of a musical event every tone can become a new fundamental tone. In this case we speak of modulation. Intervals formed by successive overtones or undertones are called epimoric ratios: (1+x)/x or x/(1+x).

Melody and harmony

Relating to succession and simultaneity respectively. Tones can be sounded either consecutively or simultaneously. In the first case we speak of melody and in the last case of polyphony or harmony.

Triads as products of epimoric ratios

The concepts of major and minor are related to the given sequences of overtones and undertones respectively. A major triad is (1+x) ... (2+x) ... (3+x). A minor triad is 1/(3+x) ... 1/(2+x) ... 1/(1+x).
An epimoric interval has both an arithmic center and a harmonic center. If we write a major triad as (x-1) ... x ... (x+1) then x is the arithmic midpoint of interval (x+1)/(x-1).
In a minor triad described as 1/(x+1) ... 1/x ... 1/(x-1), 1/x is the harmonic center of the interval 1/(x-1) : 1/(x+1) = (x+1)/(x-1).

Meantone interval

A meantone interval $x^2/(x^2-1)$ is formed by the arithmic center and the harmonic center of a given epimore interval $x(x+1)/x(x-1)$. The derivation of these formulas results from the equation of the formulas of overtone triads and undertone triads, as follows:

We multiply the minor triad by (x+1):
1 ... $(x+1)/x$ ... $(x+1)/(x-1)$

Then multiply by (x-1):$(x-1)$ ... $(x^2-1)/x$ ... $(x+1)$

Finally, multiply by x:
$x(x-1)$ ... (x^2-1) ... $x(x+1)$

We then multiply the major triad by x:
$x(x-1)$ ... x^2 ... $x(x+1)$

It follows that $x^2/(x^2-1)$ is the meantone interval of interval $x(x+1)/x(x-1)$. Moreover, this shows that a meantone interval is always an epimore. However, this does have the consequence that for even values of x the interval is not an epimore but a product of two epimores. A well-known example of this is the major sixth 5/3 (= 4/3.5/4) with the diatonic semitone 16/15 as the meantone interval.

In addition to triads of successive overtones or successive undertones, the terms major and minor can also refer to the subintervals within a triad. An epimore can be major or minor. The numerator of a major interval is always odd, that of a minor interval is always even. And with the denominators this is of course the other way around.

In principle it is also possible to consider a sequence of more than three epimores, and we then speak of tetrads, pentads, and so on. And here too the concepts of major and minor apply depending on whether we assume overtones and undertones respectively.

For x = 1, 1/0 is the meantone interval of interval 2/0, in both cases ∞ . Inaudible, beyond the reach of human hearing, symbolic of the Unknowable, the omnipotent "key note" of the Universe. For x = 2 we see the product of two epimores, viz. 2/1 and 3/2, with 4/3 as the meantone interval. For x = 3 the Pythagorean whole tone is 9/8 meantone interval of the epimore 2/1, the octave. And so forth, until at increasingly higher x-values the meantone interval becomes relatively smaller compared to the epimore interval, approaching the value 1 = geometric mean.

Sound clusters

Consisting of harmonies of at least more than three notes. In this case, music can go beyond 3-limit, 5-limit and 7-limit. In the case of 11-limit, 13-limit, etc., the music moves outside the boundaries of usual scales, which are a maximum of seven tones. Sound clusters can consist of successive overtones or undertones, and are then referred to as spectral chords.

TONALITY

Epimore cloud

Tonality is given by the presence of clusters of successive overtones or undertones, either in succession or simultaneously. We can imagine an epimore cloud around a tonal center. An epimore cloud consists of tones that have an epimore relationship to a given central tone or tonic. We can imagine a vectorial direction from a fundamental tone for each prime number, in which interval stacking is conceivable. For the number 2 these are octaves, for the number 3 fifths, for the number 5 thirds, etc. A two-dimensional cross-section of an epimore cloud is the matrix, which is 5-limit and is built up according to three vectorial directions, those angles of 60 degrees with each other. The fifths are depicted horizontally. The meantone intervals of the fifths are on lines perpendicular to the line of fifths.
Modulation is the musical dynamic in which the fundamental tone changes and thus the tonality. This also includes the interactions or resultants between these vectors. For example, from 2 and 3, in addition to fifths, also fourths follow, and from 3 and 5 both major and minor thirds follow. We can also speak of cross connections in which octavement plays a role as the first manifestation of modulation. An associated epimore cloud can be imagined around each fundamental tone.
Each tone can be the root of overtones and undertones, which are either latent or manifest.
This relativistic principle is the essence of music. Different tones are equal, and this is the secret of polyphony.

Tonic, dominant, subdominant

In connection with this concept we know the concept of scale. The framework of a scale consists of the octave interval 2/1 including the associated meantone interval, the Pythagorean whole tone 9/8. The octave is the overtone of the fundamental and the fundamental is the undertone of the aforementioned octave.

The fifth is the overtone of the root and the fourth is the undertone of the octave. The fundamental of the scale is also the root of the tonic. The fifth is the root of the dominant. And the fourth is the root of the subdominant. Overtones and undertones can be related to these three fundamental tones - a main fundamental and two sub-fundamentals: fifths and major thirds; 3-limit and 5-limit respectively. The occurrence of 7-limit intervals in a scale with no more than seven tones is a different story.
In the case of overtones, the dominant is the fifth of the tonic and the octave is the fifth of the subdominant. The fifth of the dominant produces an extra tone, namely a second, the Pythagorean whole tone 9/8, which is therefore equal to the meantone interval of the octave.
In the case of undertones, we have to think from top to bottom in terms of pitch. The fourth is the dominant relative to the octave and the fifth is the subdominant. There are therefore both major and minor versions of tonic, dominant and subdominant.

Scales

A scale is conceived within an octave and consists of a sequence of different epimores, where all tones have an epimore relationship with one fundamental tone.

A 3-limit scale, i.e. consisting of intervals that can only be described with the help of the factors 2 and 3, could only be five-tone, because from a fundamental tone in both the overtone direction and in the undertone direction only one times interval stacking is possible, which produces an epimore. This creates a total of three Pythagorean whole tones in the series through octavement, but also two intervals with the value 32/27. So there is no continuous sequence of epimores, which are always different. Therefore, there can be no question of a scale as referred to in the definition drawn up here.
A 3-limit sequence of five tones within an octave is based on a stack of fifths, which through octavement produces two consecutive Pythagorean whole tones, which is therefore also a stack that in principle refers to modulation. So in the case of a 3-limit succession of tones within an octave, there is more fragmentation than a coherent whole of epimores. In other words: 3-limit is more associated with modulation or interval stacking than with harmony around a rest point.

A pentatonic scale A C D E G A1 can also be 5-limit (factors 2, 3 and 5) in two ways:

6/5...9/8...10/9...6/5...10/9

6/5...10/9...9/8...6/5...10/9

The product of all intervals within the octave is equal to 2/1 in both cases. The intervals 6/5 and 10/9 each occur twice, but not in direct succession. The interval 9/8 (DE) turns out to be the meantone interval of A ... A1. in the second case.

If, in addition to the root tone of the octave, we also take the two mean tones of this octave as a root tone, combined with fifths and major thirds, then we can think of the seven-tone major diatonic scale C D E F G A B C1 as a result, where FG is the mean tone interval of the octave C-C1:

9/8 ... 10/9 ... 16/15 ... 9/8 ... 10/9 ... 9/8 ... 16/15
Based on undertones, we can imagine a minor diatonic scale. The order of the epimore intervals is in the opposite direction:

16/15 ... 9/8 ... 10/9 ... 9/8 ... 16/15 ... 10/9 ... 9/8

The root tone of a minor scale turns out to be the fifth of the root tone of a major scale. However, it is common practice to start a minor scale with the same root tone as that of a major scale:

9/8 ... 16/15 ... 10/9 ... 9/8 ... 16/15 ... 9/8 ... 10/9
In both cases, 9/8 is the meantone interval of the octave.
So we see that depending on which tone a scale is started with, there are different modes. There are as many conceivable modes per scale as the number of tones that make up a scale.

In both the major and minor diatonic scale and in the melodic scale we see a sequence of three epimores, namely 9/8, 10/9 and 16/15.

Just as 9/8 and 10/9 together form the interval 5/4, to 16/15 also belongs another interval. Since the numerator 16 is an even number, we know that 16/15 is a minor interval. From this follows the conclusion that the associated major interval cannot be other than 15/14, for the product of these two to be an epimore. The product 15/14 and 16/15 forms the interval 8/7. This interval is also a minor interval, which when multiplied by 7/6 forms the interval 4/3.
We are now dealing with 7-limit intervals. The diatonic semitone is therefore the gateway to 7-limit! We find the sequence of 15/14 and 16/15 in the gypsy scale. Suppose we start from the root tone C, then the fifth G forms a diatonic semitone with A flat: 16/15. Starting from the root tone, we find the following series of epimores for the gypsy scale:

9/8 ... 16/15 ... 7/6 ... 15/14 ... 16/15 ... 7/6 ... 15/14
Since until now the meantone interval was assumed to be 9/8 of the octave, we could also write the gypsy scale from G to G1:

16/15 ... 7/6 ... 15/14 ... 9/8 ... 16/15 ... 7/6 ... 15/14
We then see two fourths with the same epimoral order on either side of meantone interval 9/8.

Another angle for yet another mode is the observation that the order of 15/14 and 16/15 shows a major triad, so that the structure of the gypsy scale has apparently a major character and could therefore best correspond to this order:

5/4 (= 7/6 x 15/14) ... 6/5 (= 9/8 x 16/15) ... 7/6 ... 8/7 (= 15/14 x 16/15)
In this case the gypsy scale would start with A flat instead of C. But whether C, G, or A flat is chosen as the tonal center, in none of these cases can epimores be formed with all the notes of the scale. There always appears to be 1 tone where this is not the case.

The diatonic semitone 16/15 as found in diatonic scales gives access to 7-limit intervals because 16/15 is the minor subinterval of 8/7. Since the product of major third 5/4 and diatonic semitone 16/15 is equal to a fourth 4/3, it follows that 7/6.15/14 = 5/4. The chromatic semitone 25/24 gives indirect access to 7-limit via 13-limit, as follows: 25/24.26/25=13/12; 13/12.14/13=7/6. Furthermore, the diese 49/48 is the major subinterval of the chromatic semitone 25/24. The diese is the meantone interval of the fourth 4/3.

The harmonic scale appears to be a hybrid of the minor diatonic scale and the gypsy scale. It is obvious to also think of a hybrid tonality. However, there is only 1 fundamental tone that can form epimores with all other tones of the scale and that is the fundamental tone of the tonic.

The melodic scale is also a hybrid, namely of a major and a minor diatonic scale. Due to the simultaneous occurrence of major thirds and one minor third, the conclusion is inevitable that there is bitonality, namely the presence of two fundamental tones that together form a fifth, which in both cases are also part of the tonic. In both cases there are no unusual tones, that form no epimore with one or the other tonic.

By the way, it is true that in the diatonic scales, both major and minor, epimore relations can occur from all three tones of the tonic to all other tones of the scale.

MODULATION

Interval stacking

If the order of epimores is random, the tonality changes. There is then interval stacking in one specific vectorial direction, i.e. modulation. The audible indication of a modulation is the appearance of a leading tone.

Leading tone interval

A leading tone is primarily a tone foreign to the scale that has an epimore relationship from another tonal center with a tone that already has a relationship with the original tonic. We can call such an epimore relationship a leading tone interval.
A leading tone interval is a meantone interval of an epimore or of a product of two consecutive epimores. An example of the latter possibility is the diatonic semitone 16/15, which is the meantone interval of 5/3. If the leading tone interval is 25/24, then it is called chromaticism. This interval 25/24 is the meantone interval of the fifth 3/2 that plays a role in third modulations. The explanation for the fact that a leading tone interval is always equal to a mean tone interval lies in the fact that modulation always follows from interval stacking. If we assume an epimore interval, we assume that this interval consists of two subintervals, a major interval and a minor interval. When an interval stacking of the major subinterval takes place from the tonic of the epimore interval, this means that the minor subinterval makes way for a replica of the major subinterval. In that case, the overtone of the epimore interval is the fundamental of the meantone interval. In the case of an interval stacking of a minor subinterval, the overtone of the epimore interval is the overtone of the meantone interval. In polyphonic music, a modulation with several leading tones can take place simultaneously. For example, 9/8, 16/15 and 81/80 with a fifth modulation.

Atonal music

We could speak of atonal music as soon as music becomes multitonal. We can imagine this by two epimore clouds interpenetrating each other. This already appears to be the case with the gypsy scale. This means that instead of a successive situation as in the case of modulation, there is simultaneity here. In the case of further branching of intervals into subintervals, forms of subtonality are conceivable. In a sense, the tonality dissolves and the music loses weight, resulting in an impression of floating and timelessness.