THE FRINGE WORLD OF MICROTONAL KEYBOARDS

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Contents

Without the help of Siemen Terpstra I would not have been able to put everything I intend to publish about microtonal keyboards into just English. I want to thank Siemen for his assistance, also concerning his suggestion to add some theoretical references to mean tone tunings.

Introduction

In this treatise I want to pay attention to this issue: which minimal means are needed to express the max. Are 12 keys per octave sufficient to express tonal 5-limit music? And if not, how many keys are needed per octave? And what about 7-limit music? And so on.
The world of keyboard music instruments is indeed a fringe world, and this has always been that way throughout the centuries. A keyboard as a part of music instruments is a rational concept which is peculiar to human beings, while acoustic instruments in general reflect something of nature, that is to say the animal kingdom. A wind instument has a similarity with the vocal chords. Strings are comparable to the way crickets produce their sound. Beat instruments resemble the heartbeat and also the way some monkeys beat their chests.

A. Microtonality

At first sight it comes to microtonality when intervals are smaller than the "semitone", which can be defined as the tempered semitone in 12 e.t. Here the criterion is a keyboard instrument with 12 keys per octave, apart from the tuning system that is applied.
But let us assume that music from the renaissance period is played on a 31-tone organ. In this case it is true that the 31-tone organ is a microtonal instrument, but our perception is that this music is not microtonal. This leads us to the music idiom as a possible criterion with relation to microtonality. Then it is obvious that we define music up to and including 5-limit as non-microtonal and from 7-limit (to infinite) as microtonal. In a 7-limit music idiom we find the diesis (36/35, 49/48, 64/63) as a micro-interval, but also 7/6 and 8/7, and these intervals are definitely larger than the "semitone", but not accurately playable on a 12-tone keyboard instrument. After all, 7- limit (and 11-, 13- and so on) music can not be accurately played by a 12-tone keyboard instrument. So, the shift is between 5- and 7-limit.
Lastly, we can consider tuning systems with *tempered* intervals. It is a fact that the quotient (pure interval) / (tempered interval) is equal to a micro-interval. Based on this given it is possible to define any tempered tuning system as being microtonal. But this is a mere theoretical consideration that doesn't play an important role in musical practice.
Thus we can distinguish the following definitions of the concept "microtonality":
- 1. **Keyboard instruments** with more than 12 keys per octave
- 2. Extension of **music idiom** from (3-,) 5-limit to 7- (11-, 13- and so on) limit.
- 3. All **tuning systems** that *deviate* from just intonation, whether they are non-cyclic or cyclic, irregular or regular.

B. Just Intonation

Definitions and deductions; intervals and mutual coherence

Pure intervals are formed by the ratio's of frequencies, which can be described by simple whole positive numbers. From number 1 as point of departure we are able to arrange these whole positive numbers in a multidimensional continuum existing of vectorial directions of prime numbers. Every direction exists of an exponential row of intervals, i.e. interval stacks: $(a/b)^n$. When we assume b = 1, we achieve: $(a/1)^n = a^n$

We are able to investigate interval relations by two-dimensional sections.
As a first plan prime number 2 on horizontal axis:

9
3 6 12
1 2 4 8 16

Fig. a

From number 1 using as departure number 3 is the following number in this order of whole numbers forming a new vectorial direction. This configuration of numbers can also be depicted as follows:

8
4 12
2 6 18
1 3 9 27

Fig. b

Figures a and b are depicted by a logarithmic scale, i.e. equal distance in case of involution. Considering two triangles adjacent to each other in a 2-dimensional section (matrix) of the infinite universe of whole numbers, as shown in fig. a and fig. b, we discover that the product of the numbers on the communal side of the triangles is equal to the product of the numbers of the angles which are on a line perpendicular to the communal side. In the matrix this thesis is valid for any pair of mutual identical triangles with one communal side, i.e. in case of two triangles which are explainable by a 180 degrees rotation.
N.B.: It is also possible to consider 3-dimensional sections when 3 perpendicular dimensions represent vectors of prime numbers, for example 3, 5 and 7. Prof. A.D. Fokker made use of this (according to Euler); the octave is omitted and fifths / fourths, major thirds / minor sixths, minor thirds / major sixths, and the intervals related to prime number 7 can be depicted graphically.
The internal connections between intervals also can be shown by the following genealogical tree based on the octave interval, by which the division of intervals by arithmetic mean is apparent.

01 : 02
02 : 03 : 04
04 : 05 : 06 : 07 : 08
08 : 09 : 10 : 11 : 12 : 13 : 14 : 15 : 16
16 : 17 : 18 : 19 : 20 : 21 : 22 : 23 : 24 : 25 : 26 : 27 : 28 : 29 : 30 : 31 : 32

etcetera

Fig. c

There also exists a tree based on the octave, by which the division of the intervals by another mean than the arithmic one can be shown.

$$\tfrac{1}{2} : 1$$
$$\tfrac{1}{4} : 1/3 : \tfrac{1}{2}$$
$$1/8 : 1/7 : 1/6 : 1/5 : \tfrac{1}{4}$$
$$1/16:1/15:1/14:1/13:1/12:1/11:1/10:1/9:1/8$$
$$1/32:1/31: 1/30:1/29:1/28:1/27:1/26:1/25:1/24:1/23:1/22:1/21:1/20:1/19:1/18:1/17:1/16$$

etcetera

Fig. d

The means of the intervals, thus found, we call harmonic.
When we compare figures c and d, we distinguish two mutual mirrored patterns. In the first case the octave is divided in subsequently fifth and fourth, and in the second case the following order is reversed, i.e. subsequently fourth and fifth. For the division of other intervals like the fifth, the fourth, etc. the same kind of reversal is valid.
When we converge both trees, then we can conceive all intervals as being divided in two ways, i.e. by the arithmic mean and by the harmonic mean. The relation of harmonic mean and arithmic mean is also an interval. So, when we take a given interval as a starting point, we are able to derive three more intervals from this interval. This given is important when we think of an imaginary construction of a generalized keyboard, which will be discussed in section D of this treatise. This imaginary construction can be considered as derived from a projection of interval stacks in a logarithmic scale on the planimetry of a keyboard, thus expressing only pure intervals. In the case of the octave we achieve only Pythagorean intervals, 3-limit.

From a given interval or ratio three other intervals can be derived. In order to define these intervals I propose that we use already existing musicological terms and to generalize these. This means that, when we take for example the octave as the interval, we define the fifth as major and the fourth as minor, and the Pythagorean whole tone as the leading tone interval. The same when we take any other interval as a starting point:
interval = major . minor; leading tone = major / minor.

In mathematical formula:

interval = $(x^2 + x)(x^2 - x) = (x + 1)(x - 1)$
arithmetic mean = x^2
harmonic mean = $(x^2 - 1)$
major = $x/(x-1)$ or $(x^2+x)/(x^2-1)$
minor = $(x+1)/x$ or $(x^2-1)/(x^2-x)$
leading-tone-interval = $x^2/(x^2 - 1)$

Subsequently we can occupy ourselves with the question concerning how many times a leading tone interval fits within a major or a minor. Which value does the exponent of the leading tone interval have? The inductive method gives us an answer to our question:

major: x
minor: x – 1

This implies that, when the exponent exceeds the values found, the result will extend the (value of the) interval. The exponential stacks of the leading tone interval and their relation to the interval from which the leading tone interval is derived is essential to the approach of this treatise. The amount of leading tone intervals within the interval is equal to x + (x – 1) = 2x – 1. When we take the octave (= 2/1) as a starting point, the leading tone interval (= 9/8) will appear 2 . 3 – 1 = 5 times within the octave, just below 6 times! The interval $(9/8)^5$ = 1,802032470703125 …. and $(9/8)^6$ = 2, 027286529541015625…; in the first case smaller than 2/1 and in the second case bigger than 2/1.
The exact way to determine the number of leading tone intervals within an interval is equal to: log interval / log leading tone interval. The result will extend the value of 2x - 1.

The amount of times that a Pythagorean whole tone fits within an octave is: 5,8849192361711855097434806477 83… i.e. the amount of times a Pythagorean whole tone can be depicted within an octave in logarithmic scale.

Besides the conclusion that (2x – 1) leading tone intervals fit into a given interval, it has to be pointed out that the depiction of these leading tone intervals within this given interval is such that two rows can be distinguished. After all, when we consider the harmonic mean of an interval, we can define it as the starting point of a row of leading tone intervals, in which the arithmetic mean is also included; in addition the beginning of a given interval is starting point of a row of leading tone intervals. Both rows overlap each other like (roof)tiles when we depict these as a diagram on logarithmic scale (see section D Keyboards). Because of this we can define the notion “row” in the scope of a planimetry: a row is a succession of adjacent keys which indicate leading tone intervals. All other successions of adjacent keys are defined as columns. Both rows and columns represent different vectorial directions in the planimetry of a keyboard.
The planimetry of a keyboard is determined by only these two givens alone, i.e. the number of leading tone intervals within a given interval and two adjacent parallel rows of keys, thus forming the succession of leading tone intervals in logarithmic scale.

A number of planimetry’s can be derived from various x values:

- x=3 octave
- x=4 major sixth
- x=5 fifth

and so on.

Just Intonation keyboard concepts

It is imaginable - but not necessarily preferable! - to make music in just intonation by generalized keyboards because planimetry's based on different x-values can be seen as sections of the multidimensional continuum of prime numbers. A planimetry with x-value 3 is based on two vectorial directions of the prime numbers 2 and 3, so 3-limit music (Pythagorean scale) can be played on a keyboard that is designed in accordance with this planimetry. The planimetry's with x-values 3, 4 and 5 are sufficient for (generalized) keyboard designs in order to express the whole range of 5-limit music in just intonation. This is valid of course when keyboards of different designs based on the x values 3, 4 and 5 are played simultaneously. This issue of just intonation keyboards reminds us of the voice harmonium of Colin Brown, the organ of Henry Ward Poole, and other concepts. These were not generalized keyboards because the keys are of different shape, though all the keys of the same shape and in the same pattern in these designs form together a regularity that is compatible with the concept of a generalized keyboard. The concept of the generalized keyboard will be elaborated in section D. So, in the just intonation keyboard designs of Colin Brown, Henry Ward Poole and others it looks like more than two dimensions coincide in one level, as if several generalized keyboards are integrated in one design. This is valid for all just intonation concepts like for example the Enharmonic Organ of Thomas Perronet Thompson, the Organ of Henry Liston, the pure scale harmonium (39-tone scale) of Harry Partch, the Semantic, a just intonation keyboard by Alain Daniélou, the Enharmonic Pipe Organ in 7-limit just intonation by Martin Vogel, the Wilson/Daoud just intonation keyboard. All these keyboard designs are still irregular.

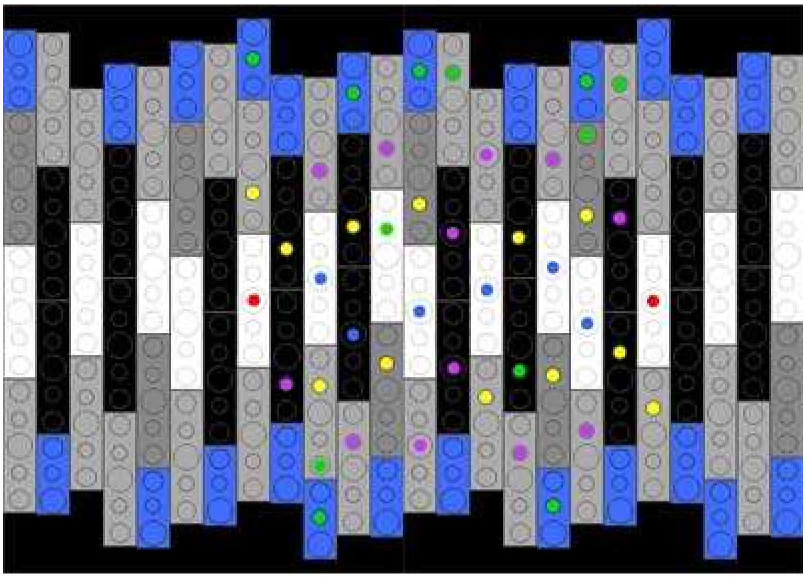

Fig. e. Harry Partch's 43-tone scale on the Tonal Plexus: it looks kind of regular, but it isn't. That is to say, this concept is not in conformity with the definition of a generalized keyboard (section D: Keyboards)

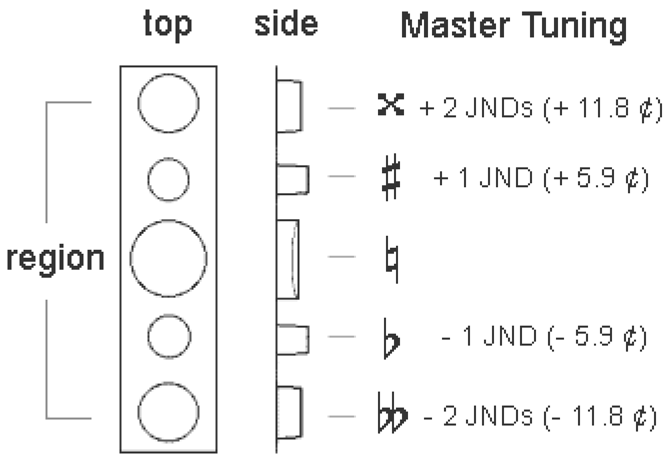

Fig. f Explanation of the pitches in Harry Partch's 43-tone scale on the Tonal Plexus

C. From geometric mean to equal temperament

Discrimination

The ability to distinguish intervals from each other by ear is closely interwoven with the concept of resolution, which I will elaborate further in section D about keyboard designs. The smaller the intervals are the more difficult they are to discriminate. So, we can imagine that the number of the small intervals that are used in musical practice is limited.

Besides the fact that the smaller the interval becomes the harder it is to distinguish, there also exists another point of view, i.e. the confusion that might arise when 3-limit intervals are compared with 5-limit intervals, 5-limit intervals with 7-limit intervals, and so on.
For example the Pythagorean third compared with the pure major third, respectively 64/81 and 64/80. The quotient of both intervals = 1,265625/1,25 = 1,0125.
Another example is the augmented major sixth compared with the 7-limit interval 7/4. The augmented major sixth we find by multiplying the major sixth with the chromatic semitone, as follows: 5/3.25/24 = 125/72 = 1,7361111. So, the quotient of the 7/4 interval and the augmented major sixth = 1,75/1,7361111 = 1,008.
When we augment a Pythagorean whole tone, like Ab-B in the harmonic scale of C, we achieve 9/8.25/24 = 1,171875. This interval is near by the 7-limit interval 7/6. The difference we find by calculating the quotient of both intervals, as follows: 1,171875/1,166666 = 1,0044648.
The differences become smaller and smaller.
There is, of course, an infinite number of these kinds of NEO's or NEA's, so to speak, when we extend music from 3-limit to 5-limit, from 5-limit to 7-limit, and so on.

Besides all this, discrimination of intervals is possible by the musical context in which intervals play a role. When, for example, chords are played, we are able to conclude that pure major thirds are meant instead of Pythagorean thirds. And so on.

Mean tone

A mean tone is the geometric mean of an interval. As soon as pure major thirds (5/4) are recognized in polyphonic music, the difference between the major whole tone (9/8) and the minor whole tone (10/9) is experienced as problematic. Especially for keyboard instruments. In order to solve this problem the mean tone temperament is developed as a compromise. Actually there exists a variety of different mean tone temperaments. The main idea in these mean tone temperaments is a line of (unequal) tempered fifths that takes the geometric mean of all major thirds, apart from the size of the major third.

Equal temperament

The idea of meantones is generalized into the concept of an equal temperament when an octave is divided to equal steps. Any interval can now be seen as the geometric mean of another interval.
The 31 e.t. as proposed by Huygens is close to the 1/4 syntonic comma mean tone temperament with regard to the pitch.
A 12 e.t. can also be seen as a meantone related temperament, i.e. 1/11syntonic comma meantone temperament.
There exist e.t.'s in which the major whole tones (9/8) and the minor whole tones (10/9) can be distinguished from each other, for example 41 e.t. and 53 e.t. These e.t.'s are not related to mean tone temperaments and they form a different family of equal temperaments.

Comparison in cents of pure intervals with tempered intervals of 12 e.t., 19 e.t., 31 e.t. and 53 e.t. in the following list:

Octave = log 2 . 1200 / log 2 =1200.
12 e.t.: 1200 cents
19 e.t.: 1200 cents
31 e.t.: 1200 cents
53 e.t.: 1200 cents

Fifth = log 3/2 . 1200 / log 2 = 701,955000865387417744370018068 14 cents
12 e.t.: 700 cents
19 e.t.: 694, 736842105263157894736842105 26 cents
31 e.t.: 696, 774193548387096774193548387 09 cents
53 e.t.: 701, 886792452830188679245283018 87 cents

Fourth = log 4/3 . 1200 / log 2 = 498, 044999134612582255513267263 44 cents
12 e.t.: 500 cents
19 e.t.; 505, 263157894736842105263157894 74 cents
31 e.t.: 503, 225806451612903225806451612 90 cents
53 e.t.: 498, 113207547169811320754716981 13 cents

Major third = log 5/4 . 1200 / log 2 = 386,31371386483481744438331153879 cents
12 e.t.: 400 cents
19 e.t.: 378, 94736842105263157894736842105 cents
31 e.t.: 387, 09677419354838709677419354839 cents
53 e.t.: 384, 90566037735849056603773584906 cents

Minor third = log 6/5 . 1200 / log 2 = 315,64128700055260030010341735063 cents
12 e.t.: 300 cents
19 e.t.: 315, 78947368421052631577894736842 1 cents
31 e.t.: 309, 67741935483870967741935483870 9 cents
53 e.t.: 316, 98113207547169811320754716981 1 cents

Second (whole tone) = log 9/8 **.** 1200 / log 2 = 203,91000173077483548897346547509 cents (major)

or log 10/9 **.** 1200 / log 2 = 182,40371213405998195540984991281 cents (minor)
12 e.t.: 200 cents
19 e.t.: 189, 47368421052631578947368421053 cents
31 e.t.: 193, 54838709677419354838709677419 cents
53 e.t.: 203,77358490566037735849056603774 cents
or 181,13207547169811320754716981132 cents

Diatonic semitone = log 16/15 . 1200 / log 2 = 111, 73128526977776481112995 cents
12 e.t.: 100 cents
19 e.t.: 126, 31578947368421052631578947368 cents
31 e.t.: 116, 12903225806451612903225806452 cents
53 e.t.: 113, 20754716981132075471698113208 cents

Chromatic semitone = log 25/24 . 1200 / log 2 = 70, 67242686428221714427989803727 2 cents
12 e.t.; 100 cents
19 e.t.: 63, 15789473684210526315789473684 2 cents
31 e.t.: 77, 41935483870967741935483870967 7 cents
53 e.t.: 67, 92452830188679245283018867924 5 cents

7/6-interval = log 7/6 x 1200 / log 2 = 266, 87090560373751118587644794125 cents
31 e.t.: 270, 96774193548387096774193548387 cents
53 e.t.: 271, 6981132075471 6981132075471698 cents

8/7-interval = log 8/7 x 1200 / log 2 = 231, 17409353087507106963681932218 cents
31 e.t.: 232, 25806451612903225806451612903 cents
53 e.t.: 226, 41509433962264150943396226415 cents

D. Keyboards

Classification of keyboard designs according to various criteria

Difference in playability

- 3- / 5- / 7-limit music polyphonically playable: just intonation in 3-limit or 12- 19- 31 e.t.; 41- 53 e.t. etc.: i.e. tuning systems based on 1 row / circle of fifths.
- 3- / 5- / 7-limit music *not* (or at least: less) polyphonically playable: multiples of 12, such as 24 e.t., 96 e.t., etc.: i.e. tuning systems based on multiple circles of fifths.

Keyscapes

- relief, for example shaped by white and black keys on 12 e.t. repetitively placed in linear arrangement as well as elevation plan as introduced by Bosanquet in case of planimetric configuration of keys (generalized keyboard)
- colour patterns in just intonation as well as in equal or unequal temperaments

Play modes in general:

- linear: 12-tone keyboard, just like flute and other wind instruments, harp
- planimetric: generalized keyboards, just like string instruments such as violin, guitar, cymbalon

Generalized keyboard

Definition: all keys are identical in shape, and the arrangement of keys follows a regular pattern.
Consequently pure tuning is out of the question, except for 3-limit for a given configuration of keys, which is possible for x = 3.
Therefore the tunings are equal or unequal tempered, with the aim of creating polyphonic music, which is 5-limit as a rule.

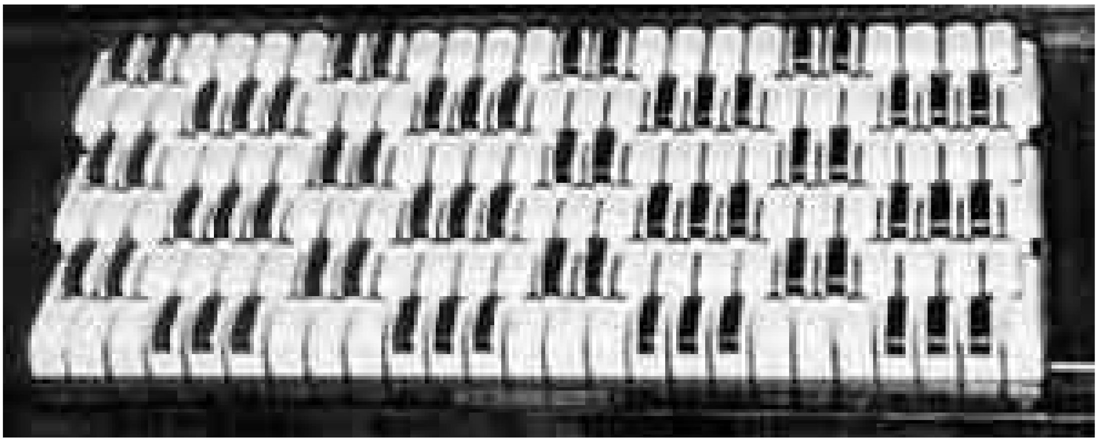

Fig. g. Von Janko Keyboard

The simplest examples are button accordion and Janko keyboard, both for 12 e.t.
Bosanquet is the founder of the generalized keyboard for tunings different from 12 e.t.

Generalized keyboards are suitable for

1. just intonation (to a limited extent) or
2. tuning systems that deviate from just intonation, i.e.
   - both unequal temperaments, like for example Werckmeister,
   - and equal temperaments.

In the vision of Gert Vos, who designed a microtonal keyboard that can be tuned in both 31 e.t. and 53 e.t., the underlying concept of a (generalized) keyboard tuned in one equal temperament is not a two-dimensional plane, a planimetry, like we have seen in section B, but the **surface of a cylinder**. When we make a drawing of one octave block in one equal temperament we have a limited number of keys which form together a group like in fig. j on page 12, fig. l on page 14, and fig. n on page 16. When we bend such a group of keys around a cylinder in such a way that the keys of the upper row touch the ones of the lower row in a logical way, we achieve a continuity of (tempered) interval stacks around a cylinder, running continuously in their specific directions round the surface of the cylinder without ending.
It is often said that equal temperaments are "cyclic" tuning systems. But this is not quite true. In the vision of Vos there are no cycles or circles, only **spirals**. One simple example can make this clear to us. When we envision a row of fifths in 12 e.t., we know that this row of twelve fifths runs through seven octaves. The row ends on the same meridian where it starts, that is to say when we envision meridians on the surface of a cylinder that run parallel with the axis of the cylinder. In the case octaves are horizontal, i.e. parallel with a meridian on the cylinder surface, like the Terpstra keyboard, octave blocks are juxtaposed parallel with the cylinder axis. On the Vos keyboard the octaves run in a spiral, which is about similar to the Fokker keyboard design.

Out of what is explained in section B about the mutual coherence of intervals, a variety of planimetry's can be constructed. The properties of each of these planimetry's leads us, much to our surprise, to the combination of equal temperaments suitable for each of these planimetry's. The specific combinations of equal temperaments on each of these planimetry's also guarantee invariance with regard to the intended intervals played on the keyboard. The finger positions will always remain the same. On the contrary, one famous example of lack of invariance is Bosanquet's generalized keyboard design. His design is recommended for the same pattern of keys as suitable for both 31 e.t. and 53 e.t., which would cause invariance to be lost, if applied simultaneously on the same planimetry, i.e. on the same keyboard.
Designs for generalized keyboards can be related to the planimetry's as sections of the multidimensional continuum of prime numbers for x-values 3, 4, 5 (and so on) as indicated in section B Just Intonation. We deduce which combination of equal temperaments is applicable for one of the planimetry´s, in such a way that any intentioned interval to be played on a keyboard will always be played on the same keys. In which case we speak of **invariance**.
For the various keyboard designs based on different planimetry´s the combinations of equal temperaments (i.e. divisions per octave) are as follows:

- x = 2: (3, 2) 5, 7, 12, 19, 31, etc.
- x = 3: (5, 7) 12, 19, 31, 50, etc. and also 43, 55...
- x = 4: (13, 9) 22, 31, 53
- x = 5: (15, 19) 34, 53, etc. and also 72 ...

and so on.

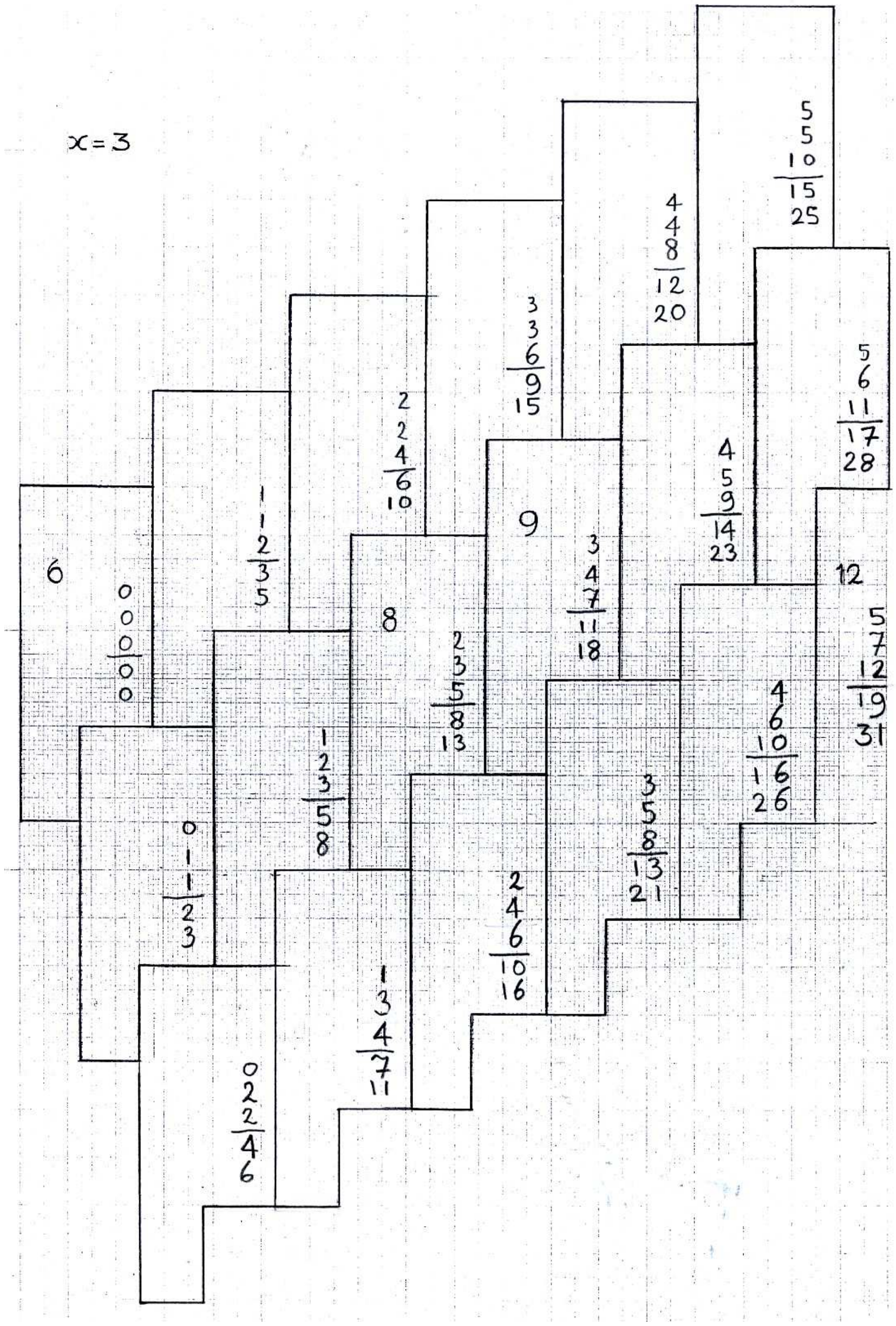

Fig. h Planimetry x = 3 including numerical indications for possible equal temperaments.

The generalized keyboard designs related to the x = 3 planimetry are by Robert Bosanquet, Adriaan Fokker, Anton van de Beer, Erv Wilson, and Siemen Terpstra.
The equal temperaments that are applicable here never make possible a distinction between the major whole tone (9/8) and the minor whole tone (10/9). So, these groups of temperaments we call mean tone related. This is even so true for 12 e.t.

Fig. i Terpstra keyboard: the first working prototype.
The hexagonal shape of the key combined with the elevation plan is evident. Terpstra's design combines features of both Erv Wilson and Robert Bosanquet.

The traditional music notation fits very well on keyboards that are designed in conformity with the planimetry with x-value 3. See fig. j below:

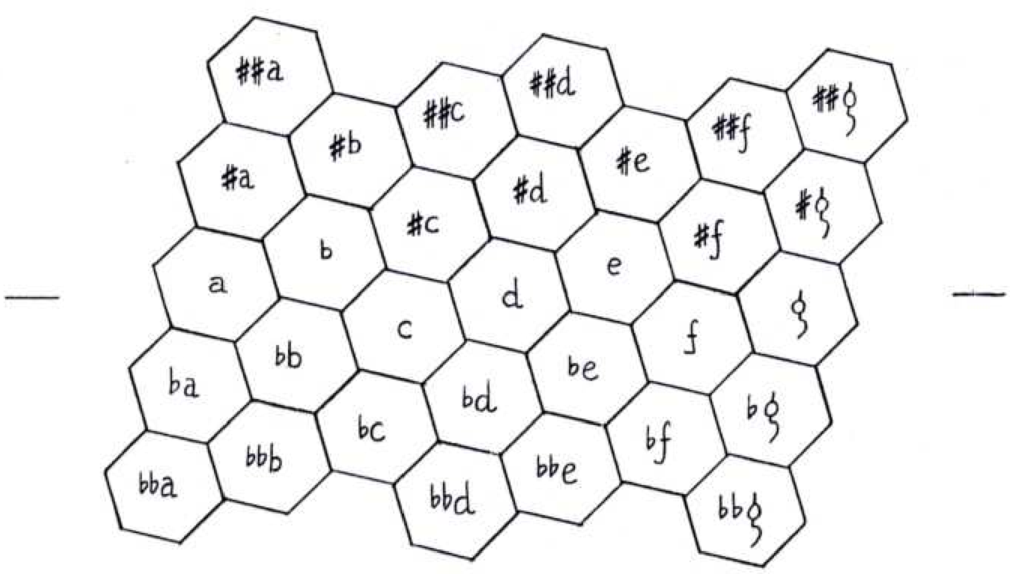

Fig. j Music notation on 31 keys per octave, suitable for: 1. Pythagorean scale in just intonation, 2. Unequal temperaments like mean tone, Werckmeister and others, 3. Equal temperaments that are mean tone related.

Music notation can also be extended as is proposed by Adriaan Fokker. In his notation system more accidentals are introduced like half sharps and half flats. This can be valuable for example in case of expressing 7-limit music idiom when all kind of weird intervals will be used. This is an area to be explored further.

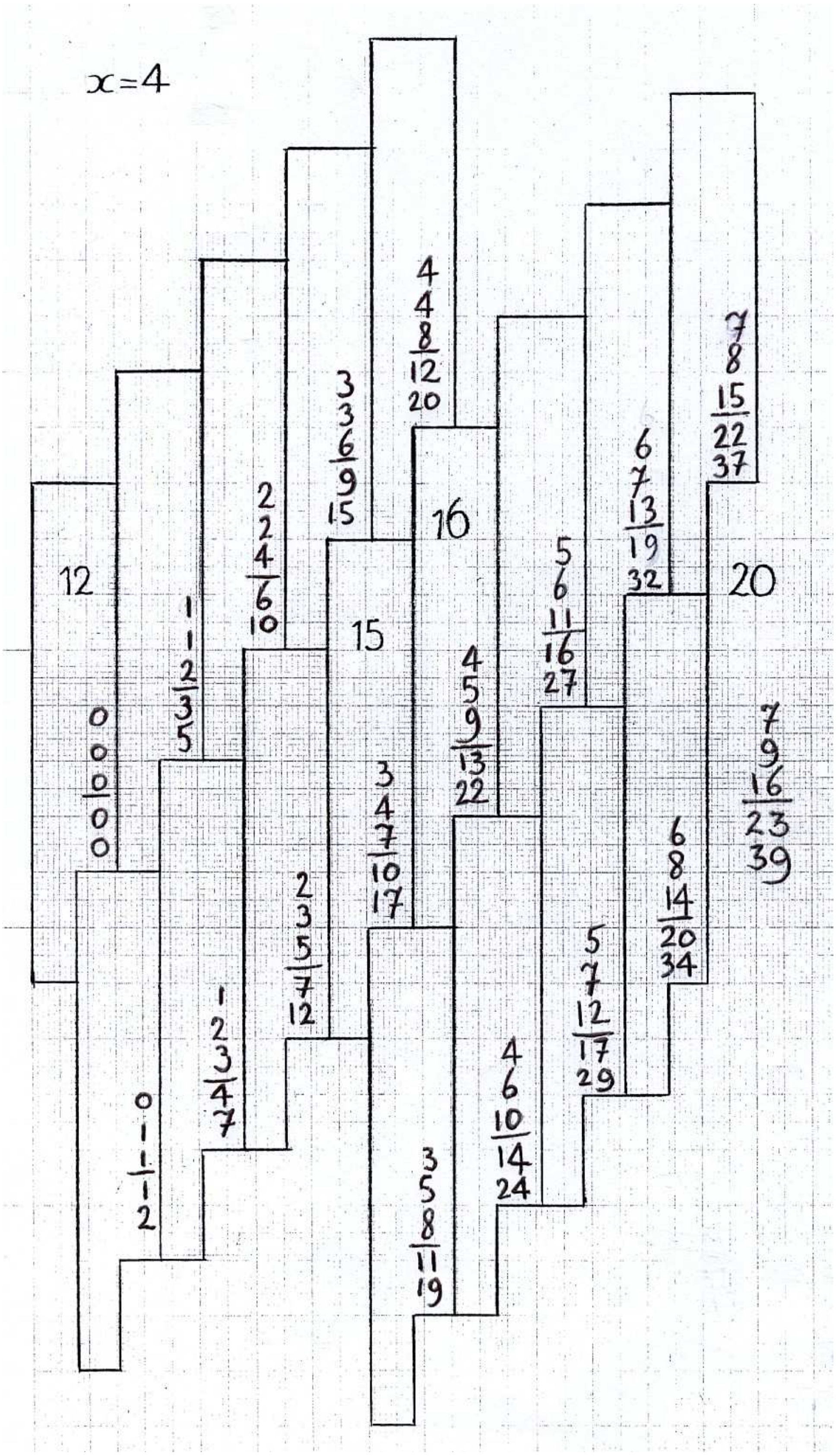

Fig. k Planimetry x = 4; the numbers indicate the possible e.t.'s within the major sixth.
Further deduction leads to octave divisions: both 31 e.t. and 53 e.t.

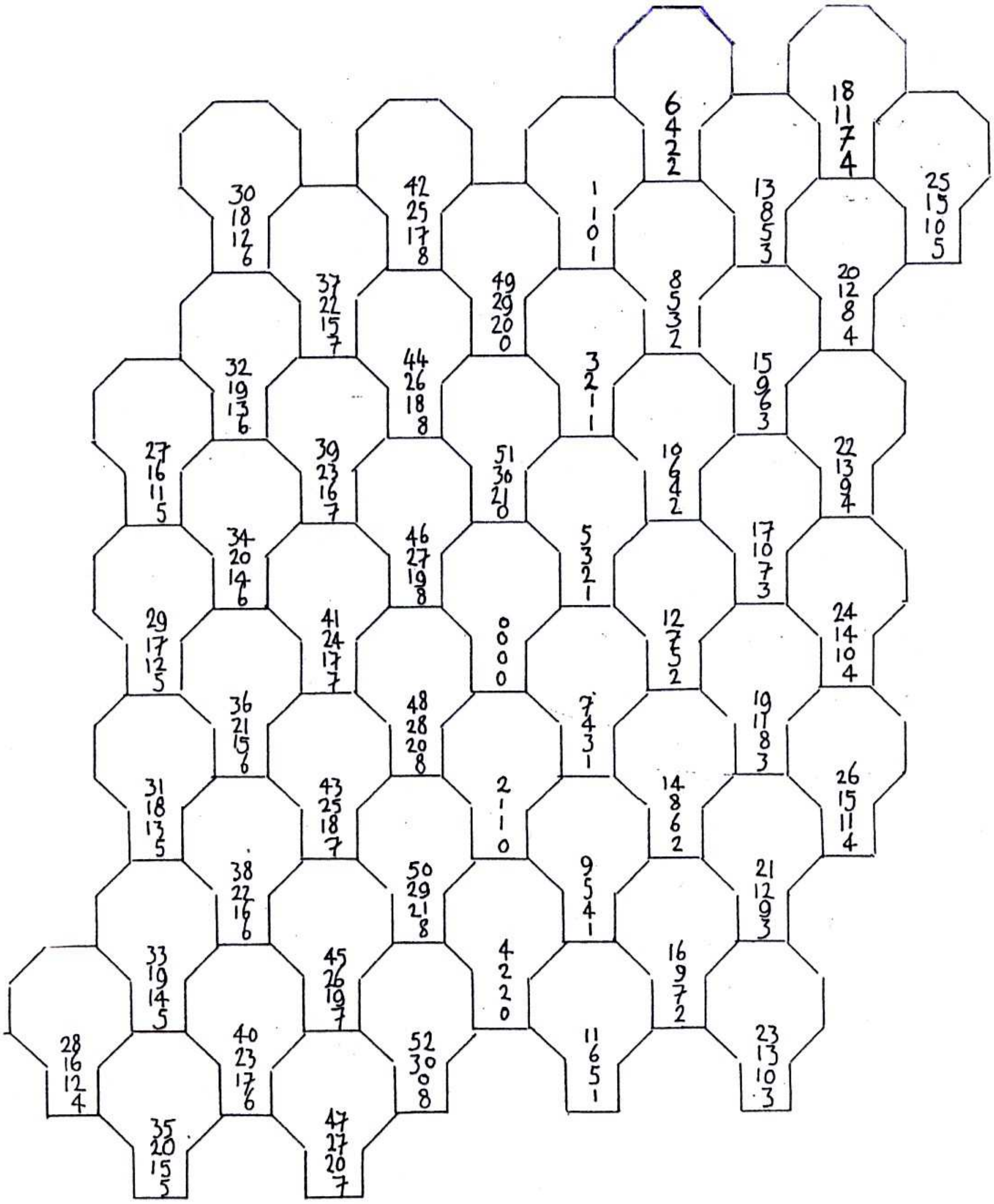

Fig. l Vos keyboard based on planimetry x = 4 (fig. k), and comparable with the plan of J.P. White for 53 e.t. as published in Helmholtz' book "Sensation of tone".

A spectacular feature of this design is the easy way to recognize the composition of a fourth by the intervals 7/6 and 8/7, only by the way the keys are arranged. In fig. l the equal steps in 53 e.t. are shown by numbers. The fourth is 22 steps. The 7/6 interval is 12 steps, and the interval 8/7 is 10 steps, together forming a diësis. Furthermore, rows of 7/6 form interval stacks $(7/6)^n$ which resemble slendro scale, and also a horizontal $3/1 \sim (7/6)^7$.

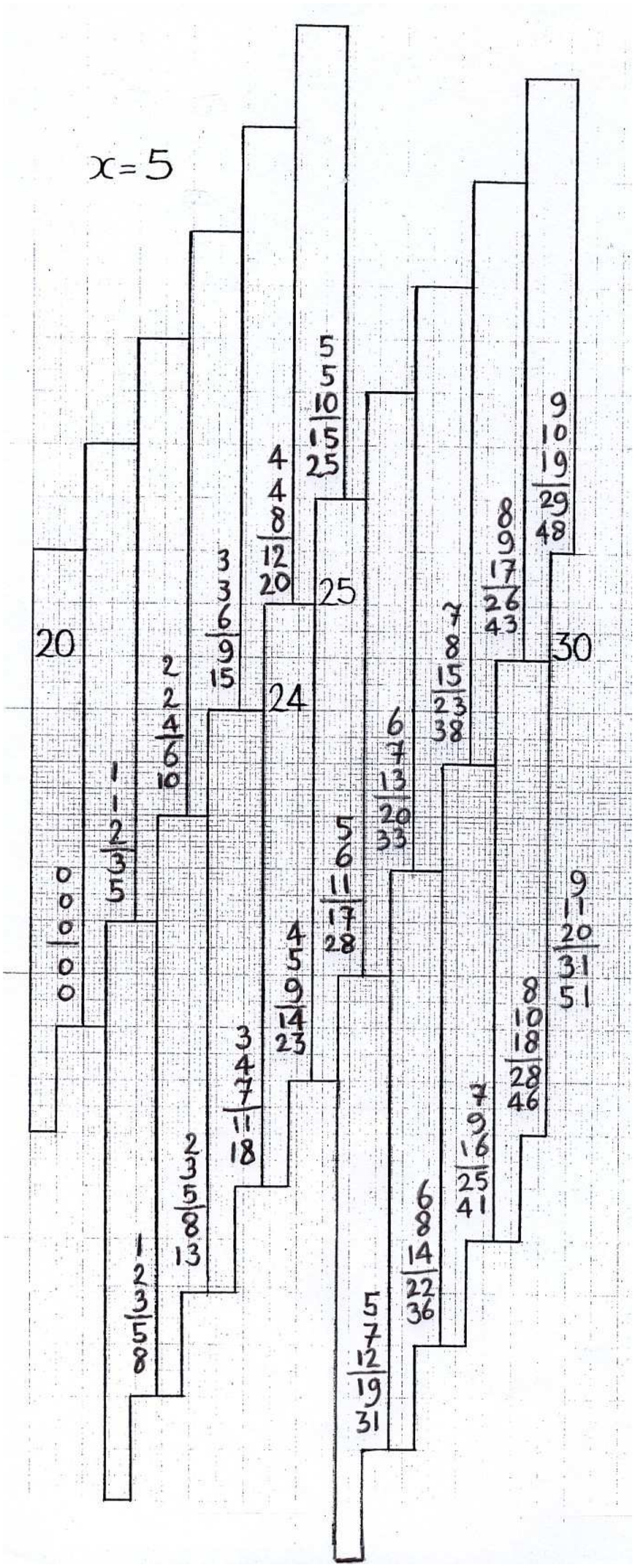

Fig. m Planimetry x = 5 enclosing the range of a fifth

The Hanson keyboard can be related to the x = 5 planimetry.

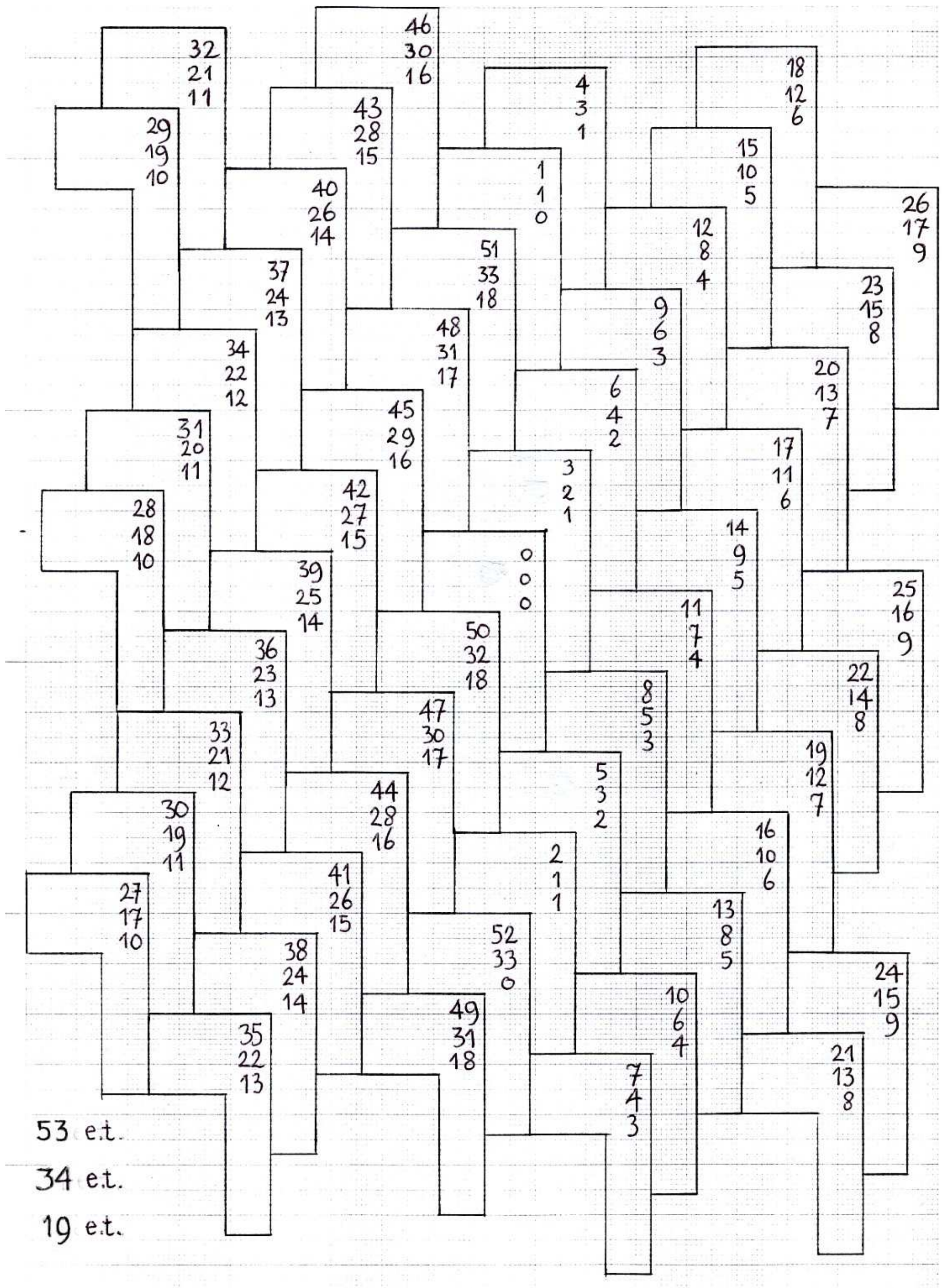

Fig. n Hanson keyboard with diagonal rows of chromatic semitones, each 3 steps in 53 e.t.

Because of the 53 e.t. tuning it is obvious that the Hanson keyboard can be compared with the Vos keyboard. It is not immediately clear if the Hanson keyboard may have some advantages above the Vos keyboard. Research and experiment (experience) will be needed.

Distinguishing key colours

The differences in key colours as we are used to are based upon tonality concepts. The white keys on a 12 e.t. keyboard express both a major scale and a minor scale. The remaining keys are black, though some organs or harpsichords have these colours exchanged. The remaining black keys form together a pentatonic scale, which refers to an original 3-limit Pythagorean tuning.

Fig. o Example of colour pattern for mean tone related temperaments on Bosanquet's keyboard design. The colours white and black indicate respectively the sharps and the flats of the keys belonging to the middle row of greenish keys. (N.B.: Green is the (symbolic) colour of the middle; bluish is distant and yellowish near by.) From top to bottom 7 – 5 - 7 - 5 – 7. This 31-tone octave block is suitable for 12 e.t., 19 e.t., 31 e.t. and eventually 43 e.t.

The most obvious solution for mean tone related temperaments seems to be a colour pattern existing of a repetitive alteration of 7 – 5 - 7 – 5 - 7, etc. Colour patterns based on the matrix model do not make sense because in mean tone related temperaments there is no comma shift.

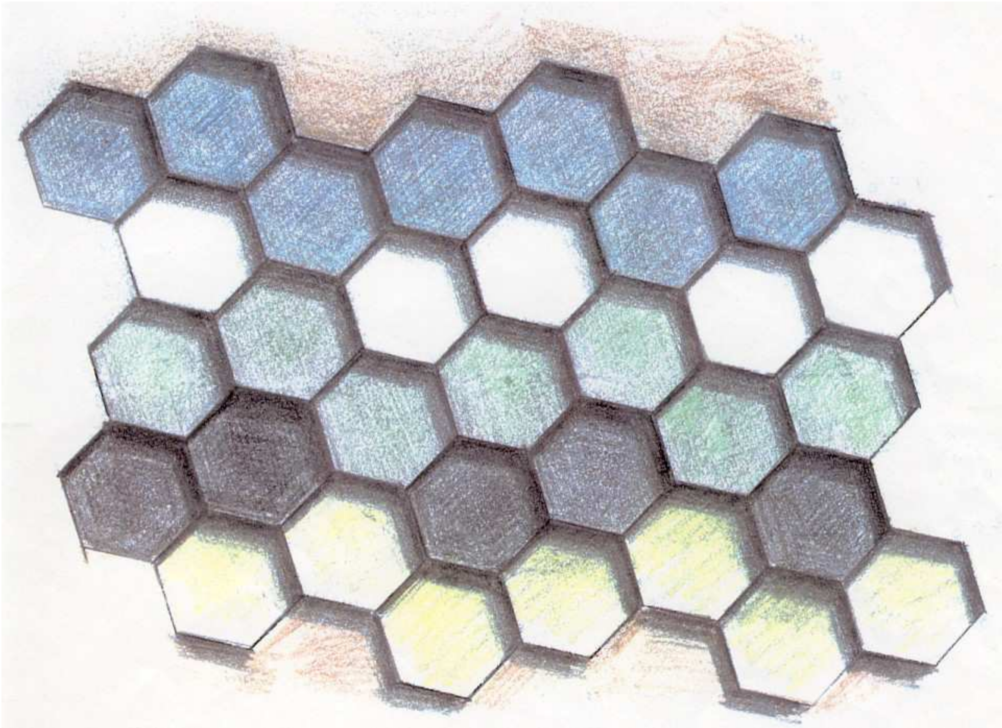

Fig. p Artist impression of one octave block of the compact keyboard of Terpstra, with same colour pattern as on fig. o. Suitable for 12 e.t., 19 e.t., 31 e.t. and 43 e.t.

Now we have to consider colour patterns for keyboards that are designed for temperaments that are not related to mean tone, that is to say temperaments by which major whole tone and minor whole tone can be distinguished from each other. As long as we pursue a colour system based on the concept of tonality we should first map tonality for 5-limit musical intervals. As we see in fig. q here below, diatonic scales, both major and minor, harmonic scale, melodic scale and gipsy scale will fit in there, but a duodenum (see fig. r) will not! A tritone from the tonal centre indicates modulation like a chromatic semitone also does. Furthermore, when we include the interval 81/80 to 5-limit tonality, the coherence will be lost because of the gaps that will appear in the matrix; so the limit appears to be the interval 25/24.

(#D)

E B #F #C

C G D A E

bE bB F C

(bD)

Fig. q 5-limit tonality around D as tonal centre, as represented in the Matrix Model. The indication of comma shifts is omitted. Because octaves are positioned orthogonal on the plane of this model fifths and fourths coincide just like major thirds and minor sixths do, and so on.

Duodenum identity by different colours of the keys can be considered in the case of temperaments like 41 e.t. and 53 e.t. in order to distinguish comma shifts. This solution is applied by Siemen Terpstra in his generalized keyboard design for 53 e.t. which is an alteration of the original design by Robert Bosanquet by inversion of the key pattern.

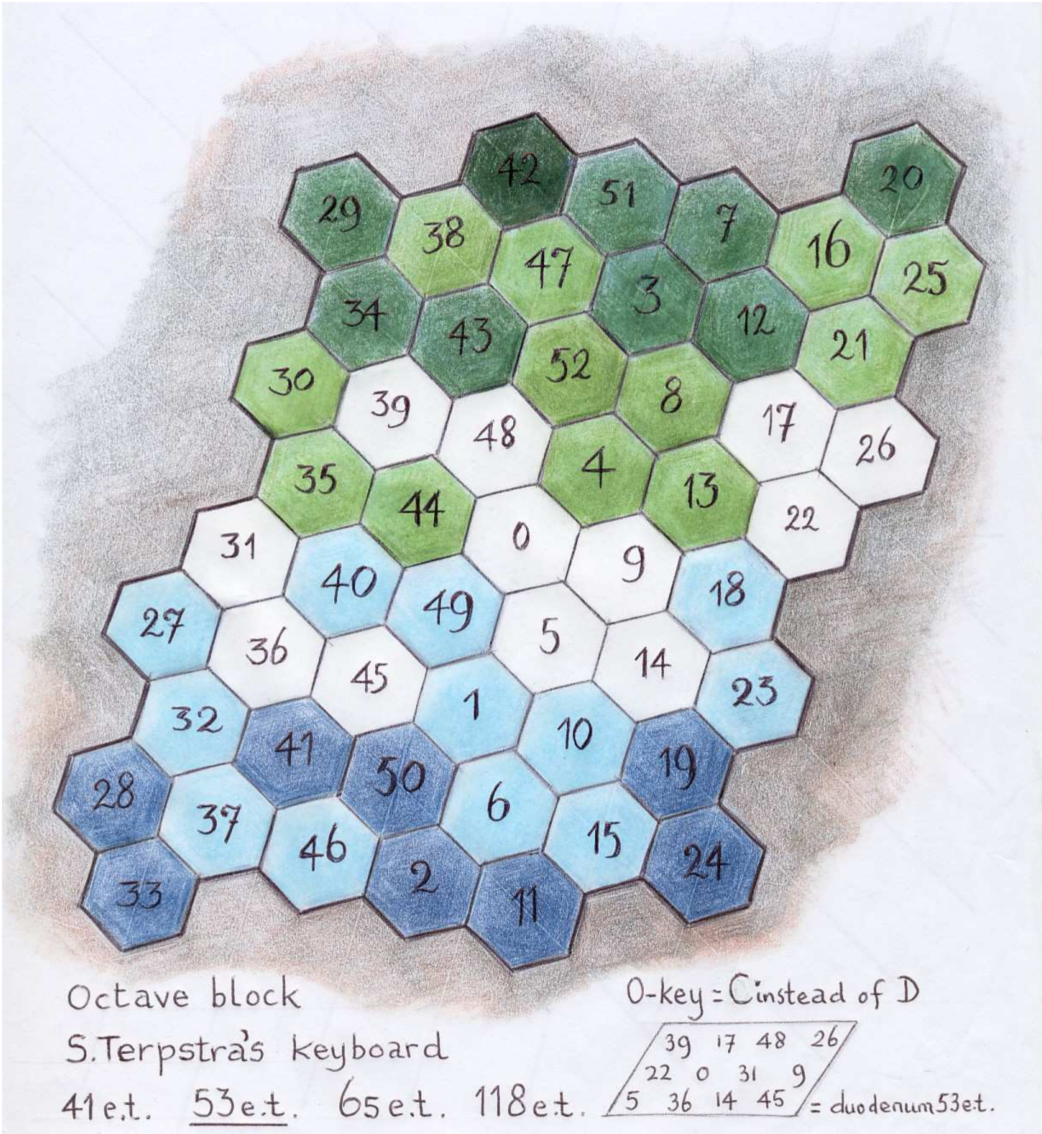

Fig. r   Octave block of Terpstra's design for 53 e.t. and related temperaments. Each colour indicates one duodenum.

The row of fifth's:
0 – 31 - 9 – 40 – 18 – 49 – **27** - 5 – 36 – 14 – 45 – 23 –
1 – 32 – 10 – 41 – 19 – 50 – 28 - 6 – 37 – 15 – 46 – 24 –
2 – 33 – 11 -
42 – 20 – 51 – 29 - 7 – 38 – 16 – 47 – 25 -
3 – 34 – 12 – 43 – 21 – 52 – 30 - 8 – 39 – 17 – 48 – **26** –
4 – 35 – 13 – 44 – 22 - 0

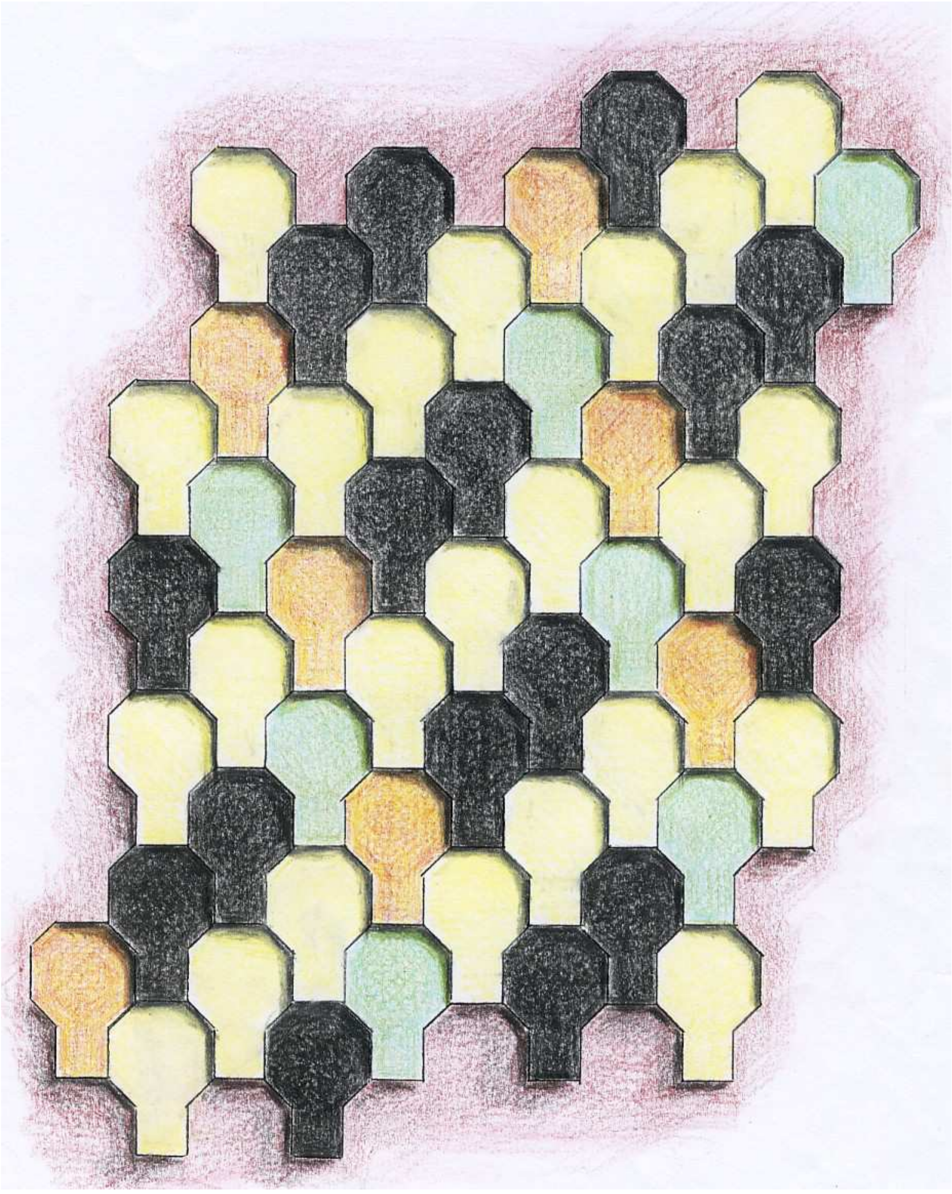

Fig. s Octave block of Vos keyboard for 22 unequal temperament (i.e. vedic tuning), 31 e.t. and 53 e.t. In this four seasons colour pattern major and minor diatonic, harmonic, melodic and gipsy scales are included by the white keys, which form three adjacent keys each time, representing a central position in the planimetry. The organisation of the coloured layers in the matrix model from top to bottom is as follows:

black
green (spring)
white (3 rows)
brown (autumn)
black

The chromatic semitone shifts from the central row of fifths in the matrix are green (sharps) and brown (flats). The remaining keys are black coloured.

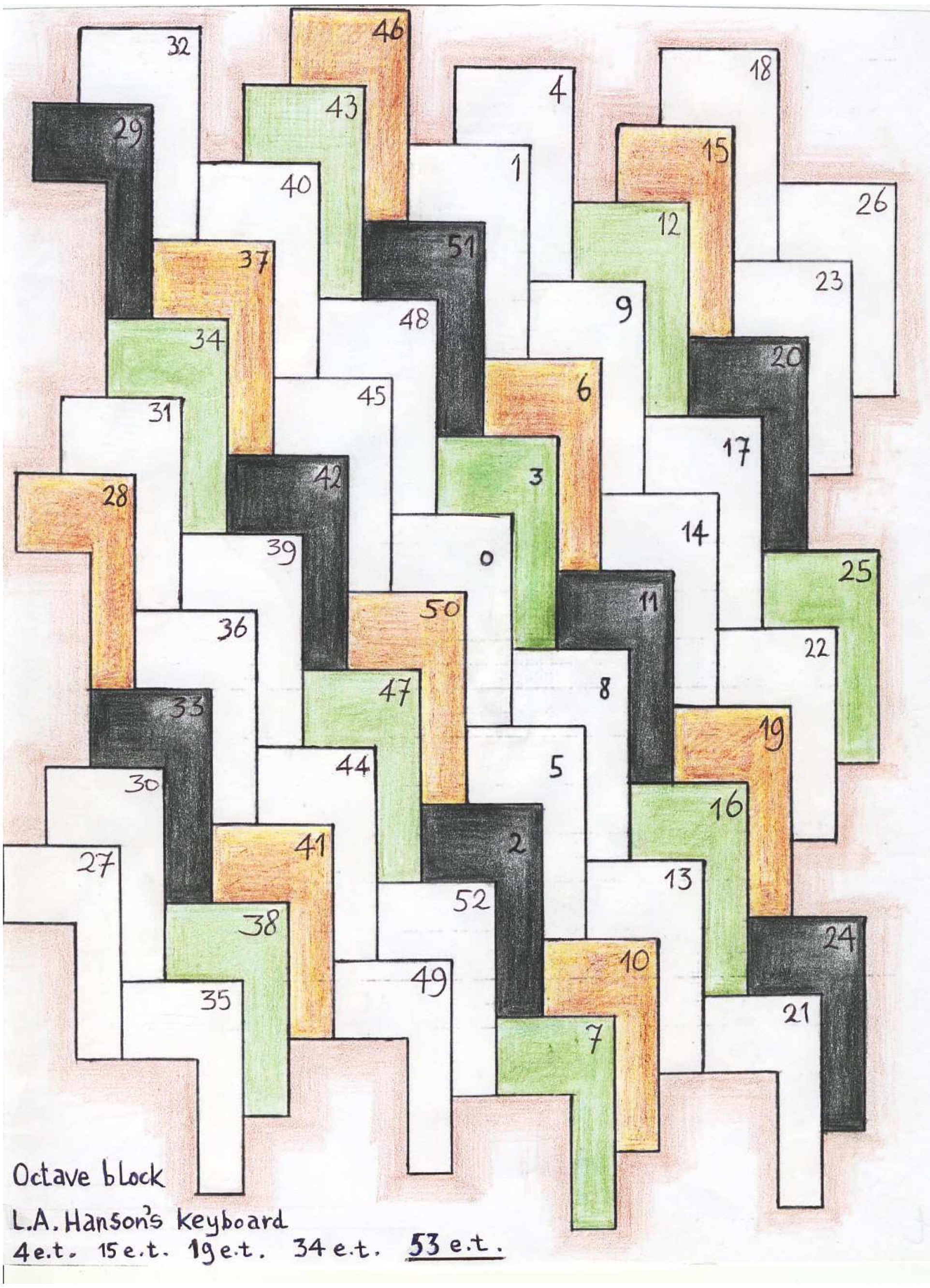

Fig. t Octave block of Hanson keyboard. The numbers indicate 53 e.t.

The colour pattern as applied on the keys is based on the same principle as on the Vos keyboard. The white keys represent the three parallel rows of fifths in the matrix model. The green and the brown keys show the chromatic semitone shifts from the middle row of fifths, and these shifts are juxtaposed in the diagonal interval stacks: $(25/24)^n$. The remaining keys are black, together forming a 3-limit structure of fifths, fourths and Pythagorean whole tones (9/8).

Comparison between keyboard designs

On the basis of criteria such as playability, compactness, inversion (in order to recognize intervals in a better way), several keyboard designs can be compared to each other. Playability will be promoted by letting the keys, which form the leading tone intervals, be adjacent to each other as much as possible.

Let us compare the designs of Fokker and Terpstra with each other. It is clear that on the Fokker organ adjacent keys form diatonic semitone, chromatic semitone and diesis. On the Terpstra keyboard for mean tone related temperaments such as 31 e.t. adjacent keys form whole tone (= mean tone), diatonic semitone and chromatic semitone.

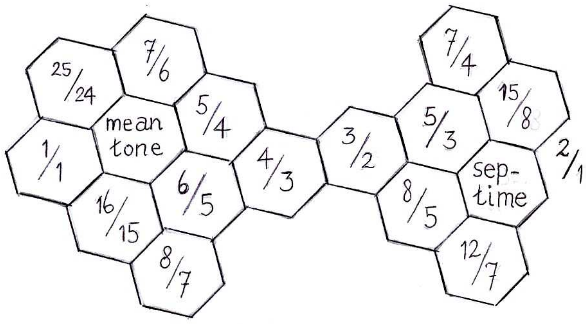

Fig. u Intervals on the Terpstra keyboard

In this aspect there exists similarity between the keyboard of Siemen Terpstra and the archiphone of Anton de Beer. However, there is a distinctive difference. The planimetry of de Beer and that of Terpstra are inverse with regard to each other. There is a good reason for this. A chromatic raising – in music notation indicated as a sharp – is on the planimetry of Terpstra actual a raising, whereas a chromatic lowering – a flat in music notation – is actual a lowering on Terpstra's planimetry in that case.

The design of Gert Vos for 31e.t. and 53 e.t. has also this property. In his design adjacent keys form diatonic semitone, diesis and also an unusual interval: 11/10 or 12/11 (a kind of three quarter tone!). In relation to the diatonic semitone, the chromatic semitone is positioned in a comparable way as on the Fokker organ, though inverse. But a chromatic semitone is not formed by adjacent keys, rather by the keys that are still pretty close at hand.

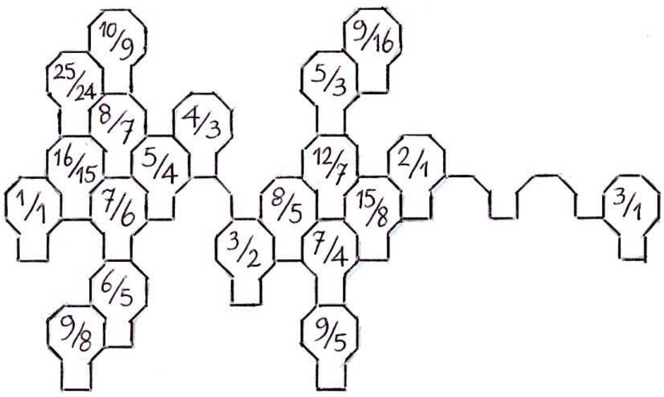

Fig. v Intervals on the Vos keyboard

There are several similarities between the designs of Fokker and Vos, particularly the key pattern and the diesis formed by adjacent keys in the columns. An advantage of the Vos keyboard is that the intervals 7/6 and 8/7 within the fourth can be distinguished much easier than on the Fokker keyboard or the Terpstra keyboard.
One of the differences is that the mean tone in 31 e.t. is defined as the geometric mean of the major third, whereas the geometric mean of the minor third is the first mean in 53 e.t. Furthermore, the possibility of 22 e.t. on the Vos keyboard refers to the applicability of vedic tuning, i.e. 22 shruti's per octave. The easy way by which a harmonic scale or a gipsy scale can be recognized in the key pattern, is connected with the stacks of diatonic semitones in the planimetry. Because of the possibility of vedic tuning, that is to say 22 unequal steps within an octave, we can conceive this keyboard design as an "Indian" keyboard.

On Hansons keyboard adjacent keys form diatonic semitone, chromatic semitone and the minor whole tone, i.e. 10/9.

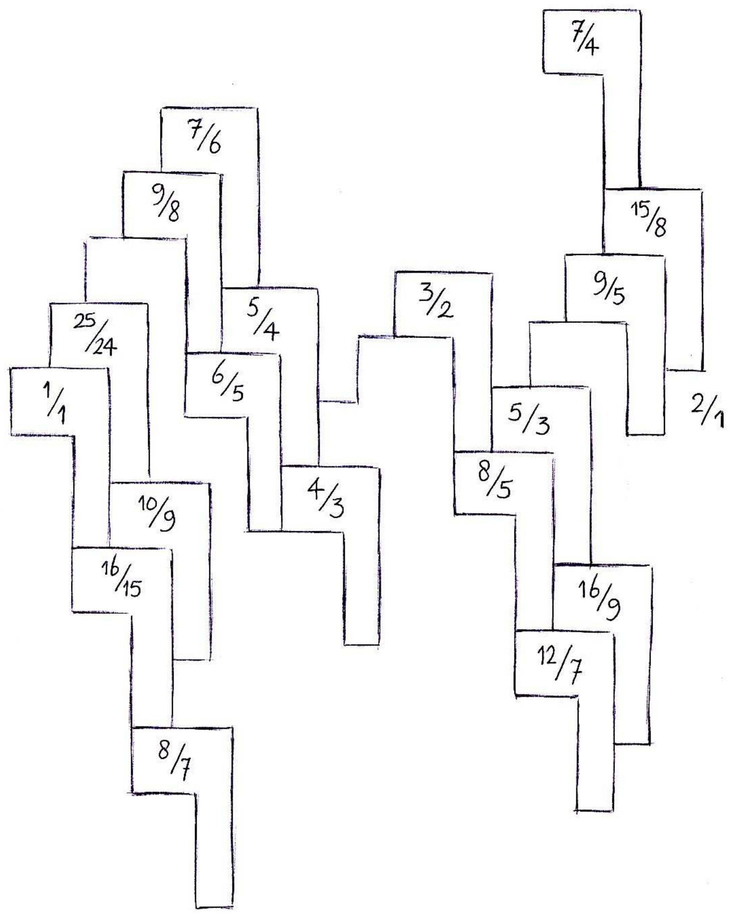

Fig. w Intervals on the Hanson keyboard

The fifths, including the major thirds and minor thirds by which the fifths are composed, are easy to find and close at hand. On the contrary, the 7-limit related intervals are not that close at hand. Within one octave four separate groups of adjacent keys form the most important intervals!

Resolution

The conception of “resolution” with regard to equal temperaments we can compare metaphorically with watching the stars without or with telescope. With help of a telescope we are able to distinguish more than with the naked eye.
In the planimetry of a keyboard which is tuned in an equal temperament some intervals can be distinguished unambiguously, and other intervals not.
Examples of ambiguity in 31 e.t.: 9/8 as well as 10/9 are represented by 5 steps (mean tone by approximation). The 3 steps which represent 16/15, also represent 15/14. The diesis represents at least 3 intervals, i.e. 36/35, 49/48 and 64/63.
In 53 e.t., as district from 31 e.t. for example, 9/8 and 10/9 can be distinguished. In this sense 53 e.t. is not a mean tone related temperament. But with regard to other intervals such as the minor third and the fourth there is actually a mean, or a geometric mean. Though there is a difference in this aspect between the minor third and the fourth in 53 e.t.: dividing these intervals in a major and a minor we find for the minor third a real mean which is 7 steps, but for the fourth we find 12 and 10 steps for respectively the 7/6-interval and the 8/7-interval.

7/6-interval = log 7/6 x 1200 / log 2 = 266, 8709056037375111858764479412 5 cents
31 e.t.: 270, 9677419354838709677419354838 7 cents
53 e.t.: 271, 6981132075471 6981132075471698 cents

8/7-interval = log 8/7 x 1200 / log 2 = 231, 1740935308750710696368193221 8 cents
31 e.t.: 232, 2580645161290322580645161290 3 cents
53 e.t.: 226, 4150943396226415094339622641 5 cents

Further investigation could possibly show us that there is a better resolution when we divide other intervals than the octave, which implies that the octaves would deviate more and more, but for 53 e.t. this is a negligible factor. For x = 4 it should be possible to divide the major sixth (5/3) in equal steps in order to investigate if some intervals would be approximated in a better way. This approach also could be valid for x = 5, etc.

When we consider the enlargement of the resolution on keyboard instruments which are tuned in an equal temperament, it is reasonable to suppose that the resolution will increase as soon as the number of steps within the octave also increases. When we prefer to restrict ourselves to monophonic or duophonic music while playing a keyboard instrument, octave divisions based on multiples of 12, like 24 e.t., 72 e.t., 96 e.t. will be sufficient. However, octave divisions by multiples of 12 will definitely be insufficient for the performance of polyphonic music!
Octave divisions which are suitable to the performance of *polyphonic* music on a keyboard instrument have to be deduced from correspondences between various interval complexes in *just intonation*, depicted in logarithmic scale. From this approach especially 12 e.t., 19 e.t., 31 e.t. and 53 e.t. result. The development of (portable) microtonal polyphonic synthesizers will contribute to the extension of the various possibilities to perform “extended harmonies” (spectral chords), modulating in 7-limit, vedic tuning, and so on.

In the time that Christiaan Huygens lived an octave division in 31 equal steps was a good solution, because mean tone temperament was the norm. From the 18th century however everything was starting to change. J.S. Bach, who was gifted by an extremely well developed ear for music, had no preference for mean tone temperament. This fact is an important sign of future developments. The most important reason that the 31 equal temperament is not that successful until now is the fact that fifths and fourths are less pure than in the 12 equal temperament, which was starting to be applied generally since the end of the 19th century. Seen in this light it is easy to understand why R.H.M. Bosanquet introduced his generalized keyboard in 53-tone equal temperament.

Epilogue

What will be the practical use of all these microtonal keyboard designs as described and explained here? Microtonal keyboards are an optional choice for 5-limit music, but the choice of microtonal keyboards will be unavoidable for 7-limit music. Microtonal keyboards are not only suitable as digital instruments but also as string instruments like harpsichords or sitar like applications, and even as pipe organs, though in this last case only in one fixed tuning system. All keyboards that are supplied with open source software should have a display showing the number of cents when you play two different keys, in combination with arrow keys or something like a pitch wheel, in order to change the number of cents for each interval that one wishes to tune.
Towards a world of subtlety and distinction microtonal music instruments, especially microtonal keyboards, can possibly play a decisive role in the exploration of new areas of harmony.